Spectral Analysis of NBA Data

Ran Cui

Department of Mathematics
University of Maryland

Spotlight Talk, Apr 2016

Q: How to quantify player contribution to team success?

- Basketball is a team sport!

- Basketball is a team sport!
- What is a better player evaluation approach? Adjusted Plus-Minus (APM)

- Basketball is a team sport!
- What is a better player evaluation approach? Adjusted Plus-Minus (APM)
- Why stop at individual player evaluation? What about group evaluation? Representation Theory!

Overview

How It Works

Idea: Borrow techniques from algebraic signal processing. We decompose a team success signal into the contributions of player groups of all sizes, from individuals through full five player lineups.

How It Works

Idea: Borrow techniques from algebraic signal processing. We decompose a team success signal into the contributions of player groups of all sizes, from individuals through full five player lineups.

Some Notations

- Average 15 players on team roster, number them $\{1, 2, \cdots, 15\}$.
- Pick 5 players as a lineup $L = \{2, 11, 7, 9, 1\}$.
- Let X be the set of all possible lineups (unordered set of 5).
- Let $V = \{h : X \to \mathbb{R}\}$.

The Representation

- $X = \{L \mid L \text{ is a lineup }\}$.
- $V = \{h : X \to \mathbb{R}\}$

The Representation

- $X = \{ L \mid L \text{ is a lineup } \}$.
- $V = \{ h : X \to \mathbb{R} \}$

  3003 dimensional real vector space with inner product

$$\langle h, k \rangle = \frac{1}{|X|} \sum_{x \in X} h(x) k(x)$$

The Representation

- $X = \{L \mid L \text{ is a lineup }\}$.
- $V = \{h : X \to \mathbb{R}\}$
  3003 dimensional real vector space with inner product

$$\langle h, k \rangle = \frac{1}{|X|} \sum_{x \in X} h(x)k(x)$$

The representation

$$S_{15} \circlearrowright V$$

$$(\sigma \cdot h)(x) = h(\sigma^{-1} \cdot x)$$

Decompose and Extract

$$V = V_0 \oplus V_1 \oplus V_2 \oplus V_3 \oplus V_4 \oplus V_5$$

Decompose and Extract

$$V = V_0 \oplus V_1 \oplus V_2 \oplus V_3 \oplus V_4 \oplus V_5$$

Suppose f = team success function.

$$f = f_0 + f_1 + f_2 + f_3 + f_4 + f_5$$

Interpretations:

f_0: Average value of f

f_1: Pure 1st order effect. Individual player contribution to f beyond mean.

f_2: Pure 2nd order effect. Pair players contribution to f.

$\vdots$

Decompose and Extract

How to extract certain player group contribution?

Decompose and Extract

How to extract certain player group contribution?

Examples

Want to see the 5th order effect of the lineup $\{2, 11, 9, 7, 1\}$

Indicator function

$$\iota_{\{2,11,9,7,1\}} := \delta_{\{2,11,9,7,1\}}$$

Let $\iota^*_{\{2,11,9,7,1\}} := (\iota_{\{2,11,9,7,1\}} \twoheadrightarrow V_5)$, take

$$\langle f, \iota^*_{\{2,11,9,7,1\}} \rangle$$

What's the 1st order effect of player 2?

Indicator function

$$\iota_2 := \sum_{2 \in L} \delta_L$$

Let $\iota^*_2 := (\iota_2 \twoheadrightarrow V_5)$, take

$$\langle f, \iota^*_2 \rangle$$

Overview

Results

Use 2013-2014 data

Define team success function f^+ and team failure function f_-

Results

Use 2013-2014 data

Define team success function f^+ and team failure function f_-

Player	Pos	Neg	Diff
stephen curry	68.68	38.73	29.96
klay thompson	61.37	33.17	28.21
draymond green	40.68	28.88	11.81
david lee	33.73	11.95	21.78
harrison barnes	31.05	42.37	-11.32
andre iguodala	29.59	2.93	26.66
andrew bogut	-1.18	-7.21	6.03
jermaine o'neal	-10.98	-2.56	-8.42
marreese speights	-19.28	-2.45	-16.83
steve blake	-31.33	-19.40	-11.93
jordan crawford	-32.17	-17.86	-14.30
toney douglas	-39.77	-22.15	-17.62
kent bazemore	-41.18	-23.20	-17.98
nemanja nedovic	-44.30	-31.29	-13.01
ognjen kuzmic	-44.92	-31.89	-13.03

Table 6: Individual effects for 13-14 GSW.

$$68.68 = \langle f^+, \iota^*_{StephenCurry} \rangle, \ 33.17 = \langle f^-, \iota^*_{KlayThompson} \rangle$$

Results

Use 2013-2014 data

Define team success function f^+ and team failure function f_-

Player	Pos	Neg	Diff
stephen curry	68.68	38.73	29.96
klay thompson	61.37	33.17	28.21
draymond green	40.68	28.88	11.81
david lee	33.73	11.95	21.78
harrison barnes	31.05	42.37	-11.32
andre iguodala	29.59	2.93	26.66
andrew bogut	-1.18	-7.21	6.03
jermaine o'neal	-10.98	-2.56	-8.42
marreese speights	-19.28	-2.45	-16.83
steve blake	-31.33	-19.40	-11.93
jordan crawford	-32.17	-17.86	-14.30
toney douglas	-39.77	-22.15	-17.62
kent bazemore	-41.18	-23.20	-17.98
nemanja nedovic	-44.30	-31.29	-13.01
ognjen kuzmic	-44.92	-31.89	-13.03

Table 6: Individual effects for 13-14 GSW.

$$68.68 = \langle f^+, \iota^*_{StephenCurry} \rangle, \ 33.17 = \langle f^-, \iota^*_{KlayThompson} \rangle$$

Results

Use 2013-2014 data

Define team success function f^+ and team failure function f_-

Player	Pos	Neg	Diff
stephen curry	68.68	38.73	29.96
klay thompson	61.37	33.17	28.21
draymond green	40.68	28.88	11.81
david lee	33.73	11.95	21.78
harrison barnes	31.05	42.37	-11.32
andre iguodala	29.59	2.93	26.66
andrew bogut	-1.18	-7.21	6.03
jermaine o'neal	-10.98	-2.56	-8.42
marreese speights	-19.28	-2.45	-16.83
steve blake	-31.33	-19.40	-11.93
jordan crawford	-32.17	-17.86	-14.30
toney douglas	-39.77	-22.15	-17.62
kent bazemore	-41.18	-23.20	-17.98
nemanja nedovic	-44.30	-31.29	-13.01
ognjen kuzmic	-44.92	-31.89	-13.03

Table 6: Individual effects for 13-14 GSW.

$$68.68 = \langle f^+, \iota^*_{StephenCurry} \rangle, \ 33.17 = \langle f^-, \iota^*_{KlayThompson} \rangle$$

Results

Top individuals: Curry, Thompson, Green, Lee, Barnes

Fives					Pos	Neg	Diff
stephen curry	klay thompson	andrew bogut	andre iguodala	david lee	32.8	9.53	23.27
jermaine o'neal	stephen curry	klay thompson	draymond green	david lee	17.07	-5.17	22.24
stephen curry	klay thompson	andre iguodala	draymond green	david lee	13.65	2.93	10.72
stephen curry	klay thompson	andre iguodala	draymond green	harrison barnes	8.88	-3.2	12.08
stephen curry	klay thompson	andrew bogut	draymond green	harrison barnes	6.86	0.25	6.61
stephen curry	klay thompson	andrew bogut	harrison barnes	david lee	5.8	13.02	-7.22
jermaine o'neal	stephen curry	klay thompson	draymond green	harrison barnes	4.67	6.43	-1.76
stephen curry	klay thompson	andre iguodala	draymond green	marreese speights	4.64	-1.33	5.97
stephen curry	andrew bogut	andre iguodala	harrison barnes	david lee	4.39	11.4	-7.01
stephen curry	klay thompson	draymond green	harrison barnes	david lee	2.96	8.14	-5.18
jermaine o'neal	andrew bogut	andre iguodala	draymond green	david lee	2.72	0.98	1.74
stephen curry	klay thompson	andre iguodala	harrison barnes	david lee	2.67	-6.55	9.22
stephen curry	andrew bogut	draymond green	marreese speights	david lee	2.53	1.48	1.05
klay thompson	andrew bogut	draymond green	marreese speights	david lee	2.52	1.35	1.17
jermaine o'neal	stephen curry	klay thompson	andrew bogut	marreese speights	2.29	1.71	0.58
jermaine o'neal	stephen curry	klay thompson	andre iguodala	draymond green	2.08	8.11	-6.03
jermaine o'neal	stephen curry	klay thompson	andre iguodala	marreese speights	1.91	1.16	0.75
jermaine o'neal	klay thompson	andrew bogut	andre iguodala	harrison barnes	1.84	0.96	0.88
klay thompson	andrew bogut	andre iguodala	marreese speights	harrison barnes	1.8	1.35	0.45
jermaine o'neal	stephen curry	andrew bogut	marreese speights	david lee	1.78	1.33	0.45
jermaine o'neal	stephen curry	klay thompson	andre iguodala	draymond green	2.08	8.11	-6.03
stephen curry	andrew bogut	andre iguodala	harrison barnes	david lee	4.39	11.4	-7.01
stephen curry	klay thompson	andrew bogut	harrison barnes	david lee	5.8	13.02	-7.22
stephen curry	klay thompson	draymond green	marreese speights	harrison barnes	1.4	11.4	-10
jermaine o'neal	stephen curry	klay thompson	harrison barnes	david lee	-2.75	9.71	-12.46

Table 10: Lineup effects for 13-14 GSW.

Results

Top individuals: Curry, Thompson, Green, Lee, Barnes

		Fives			Pos	Neg	Diff
stephen curry	klay thompson	andrew bogut	andre iguodala	david lee	32.8	9.53	23.27
jermaine o'neal	stephen curry	klay thompson	draymond green	david lee	17.07	-5.17	22.24
stephen curry	klay thompson	andre iguodala	draymond green	david lee	13.65	2.93	10.72
stephen curry	klay thompson	andre iguodala	draymond green	harrison barnes	8.88	-3.2	12.08
stephen curry	klay thompson	andrew bogut	draymond green	harrison barnes	6.86	0.25	6.61
stephen curry	klay thompson	andrew bogut	harrison barnes	david lee	5.8	13.02	-7.22
jermaine o'neal	stephen curry	klay thompson	draymond green	harrison barnes	4.67	6.43	-1.76
stephen curry	klay thompson	andre iguodala	draymond green	marreese speights	4.64	-1.33	5.97
stephen curry	andrew bogut	andre iguodala	harrison barnes	david lee	4.30	11.4	-7.01
stephen curry	klay thompson	draymond green	harrison barnes	david lee	2.96	8.14	-5.18
jermaine o'neal	andrew bogut	andre iguodala	draymond green	david lee	2.72	0.98	1.74
stephen curry	klay thompson	andre iguodala	harrison barnes	david lee	2.67	-6.55	9.22
stephen curry	andrew bogut	draymond green	marreese speights	david lee	2.53	1.48	1.05
klay thompson	andrew bogut	draymond green	marreese speights	david lee	2.52	1.35	1.17
jermaine o'neal	stephen curry	klay thompson	andrew bogut	marreese speights	2.29	1.71	0.58
jermaine o'neal	stephen curry	klay thompson	andre iguodala	draymond green	2.08	8.11	-6.03
jermaine o'neal	stephen curry	klay thompson	andre iguodala	marreese speights	1.91	1.16	0.75
jermaine o'neal	klay thompson	andrew bogut	andre iguodala	harrison barnes	1.84	0.96	0.88
klay thompson	andrew bogut	andre iguodala	marreese speights	harrison barnes	1.8	1.35	0.45
jermaine o'neal	stephen curry	andrew bogut	marreese speights	david lee	1.78	1.33	0.45
jermaine o'neal	stephen curry	klay thompson	andre iguodala	draymond green	2.08	8.11	-6.03
stephen curry	andrew bogut	andre iguodala	harrison barnes	david lee	4.39	11.4	-7.01
stephen curry	klay thompson	andrew bogut	harrison barnes	david lee	5.8	13.02	-7.22
stephen curry	klay thompson	draymond green	marreese speights	harrison barnes	1.4	11.4	-10
jermaine o'neal	stephen curry	klay thompson	harrison barnes	david lee	-2.75	9.71	-12.46

Table 10: Lineup effects for 13-14 GSW.

Results

Top individuals: Curry, Thompson, Green, Lee, Barnes

Fives					Pos	Neg	Diff
stephen curry	klay thompson	andrew bogut	andre iguodala	david lee	32.8	9.53	23.27
jermaine o'neal	stephen curry	klay thompson	draymond green	david lee	17.07	-5.17	22.24
stephen curry	klay thompson	andre iguodala	draymond green	david lee	13.65	2.93	10.72
stephen curry	klay thompson	andre iguodala	draymond green	harrison barnes	8.88	-3.2	12.08
stephen curry	klay thompson	andrew bogut	draymond green	harrison barnes	6.86	0.25	6.61
stephen curry	klay thompson	andrew bogut	draymond green	david lee	5.8	13.02	-7.22
jermaine o'neal	stephen curry	klay thompson	draymond green	harrison barnes	4.67	6.43	-1.76
stephen curry	klay thompson	andre iguodala	draymond green	marreese speights	4.64	-1.33	5.97
stephen curry	andrew bogut	andre iguodala	harrison barnes	david lee	4.39	11.4	-7.01
stephen curry	klay thompson	draymond green	harrison barnes	david lee	2.96	8.14	-5.18
jermaine o'neal	andrew bogut	andre iguodala	draymond green	david lee	2.72	0.98	1.74
stephen curry	klay thompson	andre iguodala	harrison barnes	david lee	2.67	-6.55	9.22
stephen curry	andrew bogut	draymond green	marreese speights	david lee	2.53	1.48	1.05
klay thompson	andrew bogut	draymond green	marreese speights	david lee	2.52	1.35	1.17
jermaine o'neal	stephen curry	klay thompson	andrew bogut	marreese speights	2.29	1.71	0.58
jermaine o'neal	stephen curry	klay thompson	andre iguodala	draymond green	2.08	8.11	-6.03
jermaine o'neal	stephen curry	klay thompson	andre iguodala	marreese speights	1.91	1.16	0.75
jermaine o'neal	klay thompson	andrew bogut	andre iguodala	harrison barnes	1.84	0.96	0.88
klay thompson	andrew bogut	andre iguodala	harrison barnes	david lee	1.8	1.35	0.45
jermaine o'neal	stephen curry	andrew bogut	marreese speights	david lee	1.78	1.33	0.45
jermaine o'neal	stephen curry	klay thompson	andre iguodala	draymond green	2.08	8.11	-6.03
stephen curry	andrew bogut	andre iguodala	harrison barnes	david lee	4.39	11.4	-7.01
stephen curry	klay thompson	andrew bogut	draymond green	david lee	5.8	13.02	-7.22
stephen curry	klay thompson	draymond green	marreese speights	harrison barnes	1.4	11.4	-10
jermaine o'neal	stephen curry	klay thompson	harrison barnes	david lee	-2.75	9.71	-12.46

Table 10: Lineup effects for 13-14 GSW.